FULLY WORKED SOLUTIONS

Context 5: Physics in space

Chapter 13: Exploring the final frontier

Chapter questions

1.

$$\left(\frac{T^2}{R_{av}^3}\right)_{\text{Venus}} = \left(\frac{T^2}{R_{av}^3}\right)_{\text{Earth}}$$

$$\left(\frac{0.615^2}{R_{av\ \text{Venus}}^3}\right) = \left(\frac{1^2}{1^3}\right) = 1$$

$$R_{av\ \text{Venus}}^3 = \frac{0.615^2}{1}$$

$$R_{av\ \text{Venus}} = \sqrt[3]{0.615^2}$$

$$= 0.723\ \text{AU}$$

2. Using the same process, the completed table is as follows.

Planet	**Length of year (Earth years)**	**Average orbital radius (AU)**
Mercury	0.241	0.387
Jupiter	11.86	5.20
Saturn	29.46	9.54
Uranus	84.01	19.18
Neptune	164.79	30.06

3.

$$F = G\frac{m_J m_G}{d^2}$$

$$= \frac{(6.67\times10^{-11})(1.90\times10^{27})(1.48\times10^{23})}{(1.07\times10^{9})^2}$$

$$= 1.64\times10^{22}\ \text{N}$$

4. (a)

$$F = G\frac{m_G m_A}{d^2}$$
$$= \frac{(6.67\times10^{-11})(1.48\times10^{23})(0.250)}{(2.631\times10^{6})^2}$$
$$= 0.36\ \text{N}$$

(b)

$$W = mg \qquad \therefore g = \frac{W}{m}$$
$$= \frac{0.36}{0.25}$$
$$= 1.4\ \text{m s}^{-2}$$

5. The mass of Venus is 4.869×10^{24} kg and its radius is 6052 km.

$$g = G\frac{\text{mass}_{\text{Venus}}}{(\text{radius}_{\text{Venus}} + \text{altitude})^2}$$
$$= \frac{(6.67\times10^{-11})(4.869\times10^{24})}{(6.052\times10^{6} + 0)^2}$$
$$= 8.87\ \text{m s}^{-2}$$

6. Titan's mass is 1.35×10^{23} kg and its radius is 2575 km. Orbital altitude is 150 km and the astronaut's mass is 75 kg.

$$g = G\frac{\text{mass}_{\text{Titan}}}{(\text{radius}_{\text{Titan}} + \text{altitude})^2}$$
$$= \frac{(6.67\times10^{-11})(1.35\times10^{23})}{(2.575\times10^{6} + 1.5\times10^{5})^2}$$
$$= 1.21\ \text{m s}^{-2}$$
$$W = mg$$
$$= 75\times1.21$$
$$= 90.8\ \text{N}$$

7. (a)

$$\text{Velocity} = \frac{\text{circumference}}{\text{period}}$$
$$= \frac{2\pi r}{T}$$
$$= \frac{2\pi \times 5}{2.5}$$
$$= 12.57 \text{ m s}^{-1}$$

$$\text{Centripetal force, } F_C = \frac{mv^2}{r}$$
$$= \frac{90 \times 12.57^2}{2.5}$$
$$= 5688 \text{ N}$$

(b)

$$\text{Normal true weight} = mg$$
$$= 90 \times 9.8$$
$$= 882 \text{ N}$$
$$\text{Multiple} = \frac{5688}{882}$$
$$= 6.45$$

In other words this ride exerts an apparent g-force of 6.45. That's quite a ride!

8. Note that 230 km h^{-1} = $\frac{230}{3.6}$ m s^{-1} = 63.9 m s^{-1}.

$$\text{Centripetal acceleration, } a_C = \frac{v^2}{r}$$
$$= \frac{63.9^2}{500}$$
$$= 8.16 \text{ m s}^{-2}$$

9. The mass of the Earth is 5.97×10^{24} kg and its radius is 6380 km. Orbital altitude is 250 km.

$$v = \sqrt{\frac{Gm_E}{r}}$$
$$= \sqrt{\frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(6.38 \times 10^6 + 250 \times 10^3)}}$$
$$= 7749 \text{ m s}^{-1}$$
$$= 27899 \text{ km h}^{-1}$$

10. The mass of the Moon is 7.35×10^{22} kg and its radius is 1738 km. Orbital altitude is 110 km.

$$v = \sqrt{\frac{Gm_{\text{Moon}}}{r}}$$
$$= \sqrt{\frac{(6.67 \times 10^{-11})(7.35 \times 10^{22})}{(1.738 \times 10^{6} + 110 \times 10^{3})}}$$
$$= 1629 \text{ m s}^{-1}$$
$$= 5864 \text{ km h}^{-1}$$

11.

$$v = \sqrt{\frac{Gm_{E}}{r}}$$
$$= \sqrt{\frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(6.38 \times 10^{6} + 20000 \times 10^{3})}}$$
$$= 3885 \text{ m s}^{-1}$$
$$= 13987 \text{ km h}^{-1}$$

$$\text{Time taken for 1 orbit} = \frac{\text{Length of an orbit}}{\text{Orbital velocity}}$$
$$= \frac{2\pi r}{v}$$
$$= \frac{2\pi \times (6.38 \times 10^{6} + 20000 \times 10^{3})}{3885}$$
$$= 42664 \text{ seconds}$$
$$= 11 \text{ hours } 51 \text{ minutes } 4 \text{ seconds}$$

12. Determine initial acceleration as follows:

$$a = \frac{\sum F}{m} = \frac{(T - mg)}{m}$$
$$= \frac{3.95 - (0.0885 \times 9.8)}{0.0885}$$
$$= 34.8 \text{ m s}^{-2}$$

Initial g-force can be determined as follows:

$$\text{g - force} = \frac{g + a}{9.8}$$
$$= \frac{9.8 + 34.8}{9.8 \times 0.0885}$$
$$= 4.55$$

Determine final acceleration as follows:

$$\text{final mass} = 88.5 - 10.5 = 78.0 \ g$$

$$\text{Hence, } a = \frac{\sum F}{m} = \frac{(T - mg)}{m}$$
$$= \frac{3.95 - (0.078 \times 9.8)}{0.078}$$
$$= 40.8 \text{ m s}^{-2}$$

Final g-force can be determined as follows:

$$\text{g - force} = \frac{g + a}{9.8}$$
$$= \frac{9.8 + 40.8}{9.8}$$
$$= 5.16$$

13. (a)

$$t_v = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$= \frac{1}{\sqrt{1 - \left(\frac{0.1c}{c}\right)^2}}$$
$$= 1.005 \text{ seconds}$$

(b)

$$t_v = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$= \frac{1}{\sqrt{1 - \left(\frac{0.9c}{c}\right)^2}}$$
$$= 2.294 \text{ seconds}$$

(c)

$$t_v = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$= \frac{1}{\sqrt{1 - \left(\frac{0.99c}{c}\right)^2}}$$
$$= 7.089 \text{ seconds}$$

14. (a)

$$\begin{aligned} L_v &= L_o\sqrt{1 - \frac{v^2}{c^2}} \\ &= 20\sqrt{1 - (0.1)^2} \\ &= 19.90 \text{ m} \end{aligned}$$

(b)

$$\begin{aligned} L_v &= L_o\sqrt{1 - \frac{v^2}{c^2}} \\ &= 20\sqrt{1 - (0.9)^2} \\ &= 8.72 \text{ m} \end{aligned}$$

(c)

$$\begin{aligned} L_v &= L_o\sqrt{1 - \frac{v^2}{c^2}} \\ &= 20\sqrt{1 - (0.99)^2} \\ &= 2.82 \text{ m} \end{aligned}$$

15. (a)

$$\begin{aligned} m_v &= \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \\ &= \frac{100.0}{\sqrt{1 - \frac{(0.1c)^2}{c^2}}} \\ &= 100.5 \text{ kg} \end{aligned}$$

(b)

$$\begin{aligned} m_v &= \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \\ &= \frac{100.0}{\sqrt{1 - \frac{(0.9c)^2}{c^2}}} \\ &= 229.4 \text{ kg} \end{aligned}$$

(c)

$$m_v = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{100.0}{\sqrt{1 - \frac{(0.99c)^2}{c^2}}}$$

$$= 708.9 \text{ kg}$$

16. The answers to the previous question show that as velocity increases towards c, mass also increases. Further, once velocity exceeds 0.9c, mass begins to increase significantly. Once past 0.99c, mass increases very sharply. Before ever reaching c, mass will become infinite, preventing an object from actually reaching the speed of light.

Review questions

14. (a) $F = G\frac{m_J m_C}{r^2}$

$$= \frac{(6.67 \times 10^{-11})(1.9 \times 10^{27})(10 \times 10^{22})}{(1.88 \times 10^{9})^2}$$

$$= 3.59 \times 10^{21} \text{ N}$$

(b) $F = G\frac{m_J m_S}{r^2}$

$$= \frac{(6.67 \times 10^{-11})(1.9 \times 10^{27})(1.99 \times 10^{30})}{(7.78 \times 10^{11})^2}$$

$$= 4.17 \times 10^{23} \text{ N}$$

15. $F = G\frac{m_b m_p}{r^2}$

$$= \frac{(6.67 \times 10^{-11})(1)(0.05)}{(0.15)^2}$$

$$= 1.48 \times 10^{-10} \text{ N}$$

16. The first calculation is performed as an example. The others follow the same method.

$$g = G\frac{m_{\text{Mercury}}}{r^2_{\text{Mercury}}}$$
$$= \frac{(6.67\times10^{-11})(3.3\times10^{23})}{(2.44\times10^{6})^2}$$
$$\approx 3.7\ \text{m s}^{-2}$$

Body	Mass (kg)	Radius (km)	**g on surface (m s^{-2})**	Weight of 80 kg person there
Mercury	3.3×10^{23}	2440	3.7	323
Venus	4.9×10^{24}	6050	8.9	714
Io	9×10^{22}	1820	1.8	145
Callisto	1.1×10^{23}	2400	1.3	93

17. (a)

$$a = \frac{T - mg}{m}$$
$$= \frac{6.1 - (0.0873\times9.8)}{0.0873}$$
$$= 60\ \text{m s}^{-2}$$

$$\text{g - force} = \frac{T}{9.8m}$$
$$= \frac{4.15}{9.8\times0.0873}$$
$$= 7.0$$

(b) Final mass = 87.3 – 10.5 = 76.8 g.

$$a = \frac{T - mg}{m}$$
$$= \frac{6.1 - (0.0768 \times 9.8)}{0.0768}$$
$$= 69.6 \text{ m s}^{-2}$$

$$\text{g - force} = \frac{T}{9.8m}$$
$$= \frac{6.1}{9.8 \times 0.0768}$$
$$= 8.1$$

18.

$$F = \frac{mv^2}{r}$$
$$= \frac{0.4 \times 12.5^2}{2}$$
$$= 31.25 \text{ N}$$

$$a = \frac{v^2}{r}$$
$$= \frac{12.5^2}{2}$$
$$= 78.1 \text{ N}$$

19.

Planet	**Distance (km)**	**Time taken for light wave**	**Time taken for spacecraft**
Mercury	57.9 million	3.2 minutes	24.1 days
Venus	106.2 million	5.9 minutes	44.3 days
Earth	149.6 million	8.3 minutes	62.3 days
Mars	227.9 million	12.7 minutes	95 days
Jupiter	778.4 million	43.2 minutes	324 days
Saturn	1 423.6 million	79 minutes	593 days
Uranus	2 867.0 million	159 minutes	1195 days

Neptune	4 488.4 million	249 minutes	1870 days
Pluto	5 909.6 million	328 minutes	2462 days

20. (a) $v = \frac{s}{t} = \frac{4 \times 10^{16}\,\text{m}}{1.26 \times 10^{9}\,\text{s}} = 3.17 \times 10^{7}\,\text{m s}^{-1}$

(b) $\text{fraction} = \frac{3.17 \times 10^{7}}{3 \times 10^{8}} = 0.105 \approx 10\%$

21. $v = 3660\ \text{km h}^{-1} = 3.39 \times 10^{-6}\text{c}$

$$\begin{aligned}\text{Observed diameter} &= 3480\sqrt{1 - (3.39 \times 10^{-6})^2} \\ &= 3479.99999998\ \text{km}\end{aligned}$$

22.

$$t_v = \frac{t_o}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$16\ \mu\text{s} = \frac{2.2\ \mu\text{s}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$\therefore v = 0.99\ \text{c}$$

23. <Insert figure 13A.6>

24. (a)

$$\begin{aligned}a &= \frac{T - mg}{m} \\ &= \frac{400\ 000 - (32\ 000 \times 9.8)}{32\ 000} \\ &= 2.7\ \text{m s}^{-2}\end{aligned}$$

$$\begin{aligned}\text{g - force} &= \frac{T}{9.8m} \\ &= \frac{400\ 000}{9.8 \times 32\ 000} \\ &= 1.28\end{aligned}$$

(b) Final mass = 32 000 $\times$0.15 = 4 800 kg

$$a = \frac{T - mg}{m}$$
$$= \frac{400\ 000}{4\ 800}$$
$$= 83.3\ \mathrm{m\,s^{-2}}$$

$$\text{g - force} = \frac{T}{9.8m}$$
$$= \frac{400\ 000}{9.8 \times 4\ 800}$$
$$= 8.5$$

25. (a)

$$v = \sqrt{\frac{Gm_E}{r}}$$
$$= \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(6380 + 8.8) \times 10^3}$$
$$= 7\ 895\ \mathrm{m\,s^{-1}} = 28\ 421\ \mathrm{km\,h^{-1}}$$

(b)

$$T = \frac{2\pi r}{v}$$
$$= \frac{2\pi(6388.8 \times 10^3)}{7895}$$
$$= 5085\ \mathrm{s}$$
$$\approx 85\ \mathrm{min}$$

(c)

$$F = \frac{mv^2}{r}$$
$$= \frac{0.25 \times 7895^2}{6388.8 \times 10^3}$$
$$= 2.44\ \mathrm{N}$$

26. (a)

$$v = \sqrt{\frac{Gm_E}{r}}$$
$$= \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(6380 + 180)\ x\ 10^3}$$
$$= 7791\ \mathrm{m\,s^{-1}} = 28\ 050\ \mathrm{km\,h^{-1}}$$

(b)

$$\begin{aligned} T &= \frac{2\pi r}{v} \\ &= \frac{2\pi(6380 + 180) \times 10^3}{7791} \\ &= 5290 \text{ s} \\ &\approx 88.1 \text{ min} \end{aligned}$$

(c)

$$\begin{aligned} a &= \frac{v^2}{r} \\ &= \frac{7790^2}{(6380 + 180) \times 10^3} \\ &= 9.25 \text{ m s}^{-2} \text{ toward Earth's centre} \end{aligned}$$

(d)

$$\begin{aligned} F &= ma \\ &= 110000 \times 9.25 \\ &\approx 1\,020\,000 \ N \end{aligned}$$

27. The calculation for Mars is shown.

$$\begin{aligned} \frac{r_E^3}{T_E^2} &= \frac{r_M^3}{T_M^2} \\ \therefore T_M^2 &= \frac{r_M^3 T_E^2}{r_E^3} \\ &= \frac{(58.5 \times 10^8)^3 (1)^2}{(1.50 \times 10^8)^3} \\ &= 0.059 \\ \therefore T_M &= \sqrt{0.059} \\ &= 0.244 \text{ Earth years} \end{aligned}$$

The results for the remainder of the question are shown in this table:

Planet	**Radius of orbit ($\times 10^6$ km)**	**Time to orbit Sun (in Earth years)**
Mercury	58.5	0.244
Venus	109	0.619
Mars	229	1.89

Jupiter	780	11.9
Saturn	1430	29.4

28. (a)

$$L_v = L_o\sqrt{1 - \frac{v^2}{c^2}}$$
$$80 = 120\sqrt{1 - \frac{v^2}{c^2}}$$
$$\therefore v = 0.745c$$

(b)

$$L_v = L_o\sqrt{1 - \frac{v^2}{c^2}}$$
$$= 80\sqrt{1 - 0.745^2}$$
$$= 53.4 \text{ m}$$

Jake thinks 'Oh, no! The plane won't fit!'

(c) This apparent conflict arises from the relativity of simultaneity. The nose of the plane touching the end wall of the hangar is event A. The door closing behind the plane is event B. Jock, standing on the ground, judges event B to occur before event A, while Jake, in the plane, judges event A to occur first.

29. (a)

$$L_v = L_o\sqrt{1 - \frac{v^2}{c^2}}$$
$$= 30\sqrt{1 - 0.3^2}$$
$$= 28.6 \text{ m}$$

(b)

$$t_v = \frac{t_o}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$
$$= \frac{10}{\sqrt{1 - 0.3^2}}$$
$$\therefore v = 10.483 \text{ hr}$$

difference = 10.483 - 10

= 0.483 hr = 28 min 59 s

31.

	Distance (ly)	Contracted distance (ly)	Time taken (= contracted distance / *v*)
Pluto	6.1836×10^{-4}	2.7647×10^{-5}	2.7675×10^{-5} yr = 15 min
Proxima Centauri	4.2239	0.1889	0.1890 yr = 69 days
Sirius	8.6433	0.3864	0.3868 yr =141 days
Alpha Crucis	521.95	23.336	23.36 yr
Andromeda	2.53×10^{6}	113 000	Approximately 113 000 yr